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Econometrica, . 86, N . 1 (J , 2018), 263 288

Econometrica, Vol. 86, No. 1 (January, 2018), 263–288

IDENTIFYING PREFERENCES IN NETWORKS WITH BOUNDED DEGREE

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A

((2014), M (2016), L (2015)).[2] (. ., . A (C . (2010), M (2017), B (2013)).[3] C J (2014) . A , (..,) (C (2017), G (2017), D (2014)).

B , (. ., C (2006), F (2010, 2018), G (2009), E , L , (2010), C , G , (2016), M (2015)). : ( ). A J (1996) , .

2. NE K F MA I N M DEL

.[4] ,

$u_i(G, \mathbf{X}) = u(G, \mathbf{X}; \varepsilon_i)$, ... $\varepsilon_i = (\varepsilon_{ij})_{j \neq i}$. ... (...) ... $u(G, \mathbf{X}; \cdot)$ ... G ... $\mathbf{X}$.

A ... 1: *Only connections up to distance* D *affect utility, and preferences are such that players will never choose more than a total of* L *links.*

... $d(i, j; G)$. I ... $D = 1$, ... (... C ... , J ... , ... (2009)). ... $D > 1$, ... $D = 2$ (... –). ... L ... (... L ...). ... N ... (...) ... (... F ...).[7] ... , ... A ... 1 ... . F ... , ... $D = 2$, ... L ... $L \times (L - 1)$ ... . I ... , ...

A ... 2: *Individuals are endowed with* $L \times |\mathcal{X}|$ *preference shocks, denoted* $\varepsilon_l(x)$, $l = 1, \ldots, L$, $x \in \mathcal{X}$, *which correspond to the possible direct connections and their characteristics. This vector of preference shocks is independent of* X *with a known distribution (possibly up to some finite-dimensional parameter). In addition, the support of* X *is finite.*

... (...) ... (... K ... (2004), M ... (2016)) ... (... C ... (2006), G ... (2009)). ... I ... , ...

[7] F ... N ... L ... A ... A ... H ... (... A ... H ... – ...), ... I ... , ... I ... J ... -B ... (2012), ... 1 ... 1,000 ... 5 ... 8 ... , ... (... 37, ... 1879).

[8]

D. I $|\mathcal{X}| \times L \sum_{d=1}^{D}(L-1)^{d-1}$ (

L >

$N(i)$ ... i (.., $N(i) \equiv \{j : G(i,j) = 1\}$) ... $|\cdot|$ ... . I ... , ... $l(j)$ ... j l ... i ... .[11] ... $(N(i))$ ... $(\{i\})$ ... . , ... L ... , ... $D = 2$, ...

A ... . (J ... - (1996)).

DEFINITION 1 ... : A ... ij ... i ... j ... , ... - ... :

$$\forall i,j: G(i,j) = 1 \quad u_i(G, \mathbf{X}) \geq u_i(G_{-ij}, \mathbf{X}) \quad \ldots \quad u_j(G, \mathbf{X}) \geq u_j(G_{-ij}, \mathbf{X}); \qquad (\;)$$

...

$$\forall i,j: G(i,j) = 0 \quad u_i(G_{+ij}, \mathbf{X}) > u_i(G, \mathbf{X}) \quad \ldots \quad u_j(G_{+ij}, \mathbf{X}) < u_j(G, \mathbf{X}) \qquad (\;)$$

I ... , G_{-ij} ... $(k,l) \rightarrow G_{-ij}(k,l) = G(k,l)$ ... $(k,l) = (i,j)$ ... $(k,l) \rightarrow G_{-ij}(k,l) = 0$ ... $(k,l) = (i,j)$. A ... , G_{+ij} ... $(k,l) \rightarrow G_{+ij}(k,l) = G(k,l)$ ... $(k,l) = (i,j)$ ... $(k,l) \rightarrow G_{+ij}(k,l) = 1$ ... $(k,l) = (i,j)$. ... ; B ... J ... (2006) J ... (2009). A ... - ... , ... , ... - ... . N ... , ...

3. E IE F E

H ... , ... (1). I ... , ... ($L = 1$, ... $D = 1$), ... : B ( ) ... W ( ). ... , ... , ... (x,y), ... , ... (x) ... $(y$, ... $y = 0$ ...). F ... , (B,W) ... *network types* ... . (M ... , ... (B,W), ...) ... , ... , (x,y). ... (1) ... $u_i(x,y) = f_{xy} + \varepsilon_i(y)$, ... f_{xy}, $x,y \in \{B,W\}$, ... ,

2 10.4608 0 1 8 0.4608 -24 322(08 J/ 11 1 -0.0 2.22 0 ()) 6.4757 0 0 6.4

( ). f_{xy}.

F (G X) *type shares*. F 500 , 50 (B W) 0.1 ((W B) 0.1,). f_{xy} .

, (.,). F : , $f_{BB} + \varepsilon_i(B) \geq 0$ $f_{BW} + \varepsilon_i(W) < 0$. H , (B W) - (B B) *preference classes*, H. F , H = {(B 0) (B B)}. (B ., (B 0), - .) : H_1 = {(B 0)}, , ; H_2 = {(B 0) (B B)}, , ; H_3 = {(B 0) (B W)}, , ; H_4 = {(B 0) (B B) (B W)}, , . W.

E , ε, - – (., H - H, H). H , - - . I ε(B) ε(W) f_{xy} . , - , M C (7 M A D.7.1 (, - , , (2017))).

G - , *allocation parameters*, $\alpha_H(\cdot)$, , H –. F , (B W), (H1|B)α_{H_1}(B W) + (H2|B)α_{H_2}(B W) + (H3|B)α_{H_3}(B W) + (H4|B)α_{H_4}(B W) (-).

. H , , , , . F , .

4. NE K E AND EFE ENCE CLA E

N . *network types.* , ( ) . D L . , (. .,), , . I , , , — — , — .

M , , A, , v. A D , *ego* . I D. 1 , 0 . L 2, L(L − 1) . 1 + L + L(L − 1) + L(L − 1)^2 + ··· + L(L − 1)$^{D-1}$ = 1 + L $\sum_{d=1}^{D}$(L − 1)$^{d-1}$.[12] v A. v , v$_1$. , $\mathcal{X}$ ∪ {0}, 0 , .[13] :

DEFINITION 2 N : F D, L, $\mathcal{X}$. A t t = (A, v), A 1 + L $\sum_{d=1}^{D}$(L − 1)$^{d-1}$ v A. - t. v . 70 310.4603

IDEN IF ING EFE ENCE

1. 2. 3. 4.

N , A 1 2, A 1 A, A 2 v, . A , $u_i(G, \mathbf{X}) = u(A, v; \varepsilon_i)$. ε_i (.., $L \times |\mathcal{X}|$). I E 1, : $\varepsilon_i = (\varepsilon_{i1}(B), \varepsilon_{i1}(W), \varepsilon_{i2}(B), \varepsilon_{i2}(W))$.[15] F F 2,

$$f(B, B) + \varepsilon_{i1}(B) + f(B, W) + \varepsilon_{i2}(W) + \omega$$

$$f(B, B) + \varepsilon_{i1}(B) + f(B, W) + \varepsilon_{i2}(W) + \nu$$

(ω ν).

I . F , *preference classes*, . . I , , . N , ε: ( )

$\varepsilon_{i2}(B)$ i

t :
$\mu_{v_1(t)} \sum_H P_{H|v_1(t)} \alpha_H(t)$, $\mu_{v_1(t)}$ $v_1(t)$
(t). t
μ.

5. IDEN IFICA I N I H NE K E

I , (, *type shares*)
, . I,
,
. , ,
. ( ):

CONDI ION 1 E L : A . F t
H, t H = $\alpha_H(t) = 0$.

CONDI ION 2 N L B D I : 2D
(d(i j; G) > 2D). F t s
L , t̄ s̄
2D ,

$$\left(\mu_{v_1(t)} \sum_{\tilde{H} \in \mathcal{H}} P_{\tilde{H}|v_1(t)} \alpha_{\tilde{H}}(t) 1_{\bar{t} \in \tilde{H}}\right) \cdot \left(\mu_{v_1(s)} \sum_{\breve{H} \in \mathcal{H}} P_{\breve{H}|v_1(s)} \alpha_{\breve{H}}(s) 1_{\bar{s} \in \breve{H}}\right) = 0$$

. I
, P.|.,
.
(.. .,)
.
$\pi \equiv (\pi_t)_{t \in \mathcal{T}}$, π_t
t.
.[17] F π
$\{P_{H|v_1}\}$

C 1 2 -

. A ,

. B -

, M A B. H ,

C 1 2

.

C 1 ( ) . I ,

,

, -

( ). A C 1 ,

. L C 1

($\alpha_H(t) > 0$ H t H),

,

C 2 ( ) . I *who are at a distance greater than 2D from each other* .

2D

(M A B).

C 2,

(t s), (t t),

L . , ($\bar{t}$ $\bar{s}$), -

t s

2D . F , t s

F 3, E 1 (D = 2 L = 2). I

2D,

$\bar{t}$ $\bar{s}$. -

( ) -

. 2D + 1,

-

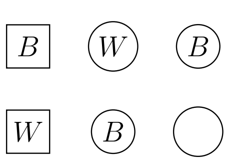
D. I ,

(2D), C 2

G.

( ) I ( )

Initial Types



( ) I ( )

Initi

t ($\alpha_{\tilde{H}}$(t) $\tilde{H}$) $\bar{t}$ ($1_{\bar{t}\tilde{H}}$). ( ), $f_{BW} + \nu + \varepsilon_2(W) \geq 0$.[18] s $\bar{s}$. I , t ( ), s ( ), . I t s $\bar{t}$ $\bar{s}$, . , . , C 1 2 t s. H , (.., t $\bar{t}$ s $\bar{s}$).

1 ( ) ( ) , C 1 2 . ( ). I , C 1 2. I , . I *no* C 1 2, . , . , .

6. IM LEMEN A ION

1 . F , ( ) .[19] , .

6.1. *Formulation as Quadratic Programming Problem*

C 2 . , 1 .

---

18 $\bar{t}$ $2f_{BW} + 2\nu + \varepsilon_1(W) + \varepsilon_2(W)$, t ($\bar{t}$) $f_{BW} + \nu + \varepsilon_1(W)$. H , $\bar{t}$ $f_{BW} + \nu + \varepsilon_2(W) \geq 0$. , $\bar{s}$ $f_{WB} + \nu + \varepsilon_2(B) \geq 0$.

19 A – H (2006) . F , K (2013).

C 1: $\{\alpha_H(t) : t \in H\}$, α . C 2 $\alpha' Q\alpha$, Q . A $(\frac{1}{\mu}\sum_H \mu_{v_1(t)} P_{H|v_1(t)}(\theta)\alpha_H(t) = \pi_t)$.

$$\min_{\{\alpha_H(t):t\in H\}} \alpha' Q\alpha \quad :$$

$$\frac{1}{\mu}\sum_H \mu_{v_1(t)} P_{H|v_1(t)}(\theta)\alpha_H(t) = \pi_t \quad \forall t; \tag{3}$$

$$\sum_{t\in H}\alpha_H(t) = 1 \quad \forall H; \qquad \alpha_H(t) \geq 0$$

A , 1 . , θ (, $\{P_{H|v_1}(\theta)\}$), , .

Q, C 2. (-) Q - $\alpha_H(t)$ $\alpha_G(s)$. 1 C 2, 0. , $Q_{[\alpha_H(t)\alpha_G(s)]} = 1_{\bar{t}\in H}\cdot 1_{\bar{s}\in G}$, $\bar{t}$ $\bar{s}$ t s ( ). $(\alpha_H(t)1_{\bar{t}\in H})\cdot(\alpha_G(s)1_{\bar{s}\in G})$ $\alpha' Q\alpha$. - , C 2 $(\mu_{v_1(t)} P_{H|v_1(t)}\alpha_H(t)1_{\bar{t}\in H})\cdot(\mu_{v_1(s)} P_{G|v_1(s)}\alpha_G(s)1_{\bar{s}\in G})$. H , $\mu_{v_1(\cdot)}$ $P_{\cdot|\cdot}$ [20] , . Q 3.

E AM LE 2: L (1), $D = 1$, $L = 1$, $\mathcal{X} = \{B, W\}$. H , $v = (v_1, v_2)$. -

P(1 Tf 0 Tc 0.969 1013j /T1_1H1 8 Tf 6.4757648 6.4757 382.127 236.365 m 382.2014 280_231.3

A , $H_1 = \{(B 0)\}$, . H , $1_{\bar{t} H_1} = 0$ $\bar{t}$. ( ) Q: $\alpha_2(B 0)$, $\alpha_3(B 0)$, $\alpha_4(B 0)$, $\alpha_6(W 0)$, $\alpha_7(W 0)$, $\alpha_8(W 0)$. . (B L = 1, .) F $\alpha_3(B 0)$, 1' $\alpha_7(W 0)$ $\alpha_8(W 0)$. $H_3 = \{(B 0)(B W)\}$, $H_7 = \{(W 0)(W B)\}$, $H_8 = \{(W 0)(W W)(W B)\}$, . , $t = (B 0)$ ( ) $s = (W 0)$ ( ), $\bar{t} = (B W)$ ( ) $\bar{s} = (W B)$ ( ), t s. , $1_{\bar{t} H_3} \cdot 1_{\bar{s} H_7} = 1$ $1_{\bar{t} H_3} \cdot 1_{\bar{s} H_8} = 1$.

Q $1_{\bar{t} H} \cdot 1_{\bar{s} G}$. G $\alpha_H(t)$ $\alpha_G(s)$, t s $\bar{t}$ $\bar{s}$, $\bar{t}$ $\bar{s}$ H G, . H , . I , Q S, Q, $S_{[\alpha_H(t) \alpha_G(s)]}$ $1_{\bar{t} H}$ $1_{\bar{t} H} \cdot 1_{\bar{s} G}$. C , S Q S , . S - - - . Q H (.,) S : $Q = S \circ S$. (M A D.7.2 .)

Q , . (A A.)

HE EM 2: *Given a probability distribution of preference classes* $\{P_{H|v_1}(\theta)\}$, *there exists a vector of allocation parameters* α *yielding type shares* $\{\pi_t\}$ *while satisfying Conditions* 1 *and* 2 *if and only if the optimal value of QP problem* (3) *is zero*.

. B , , . I , , .[21] C 1 2.

21

F , $\alpha' Q \alpha$ Q , .[22] I 7, KNI . I (.., $\alpha' Q \alpha = 0$), .

6.2. *Consistency of Type Shares*

, θ, . 1 2 , *knowing the type shares* π, θ. π

A (π),

. H , . I , ( ) . G (1961), [23]

.745 0 (),)34411 1 .6(A)19(,6( )6 0 ( ))-70 0 21 20() / 10 1

I ION 1: *Under random sampling* (*on the underlying*,

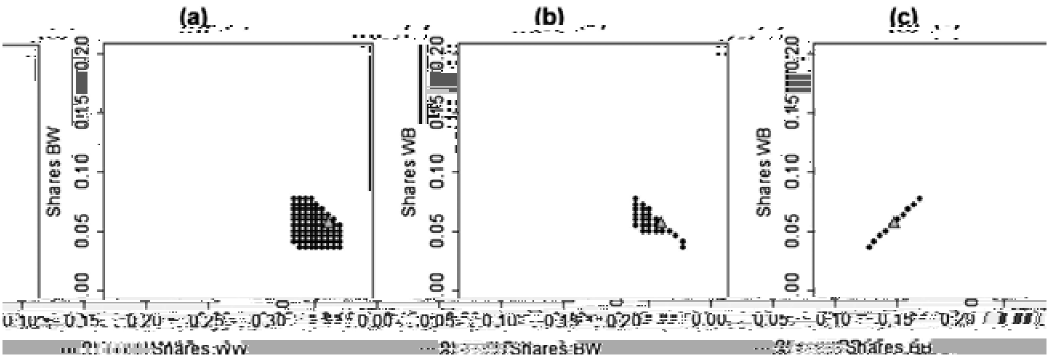


FIGURE 5. E [illegible] *Notes*: [illegible] f_{BB}

FIG 6. L ... *Notes*: ... (... F 5). D ... : $f_{BB} = 0.40$, $f_{BW} = 0.20$, $f_{WB} = 0.15$, $f_{WW} = 0.50$.

... (... M A B).[29] ... (...). ... , (A v), ... (... M A D.3).

... , ... $n = 500$ (100 ... 400 ... , ... $\mu_B/\mu_W = 1/4$), ... M A D.4. F 7 ... , ... , ...

[29] ... $(f_{BB}, f_{BW}, f_{WB}, f_{WW}) = (-0.9, -1.5, -1.7, -0.7)$, $\nu = 0.2$, ... $\omega = 0.2$. F ... D.7 ... M ... . A. H ... .

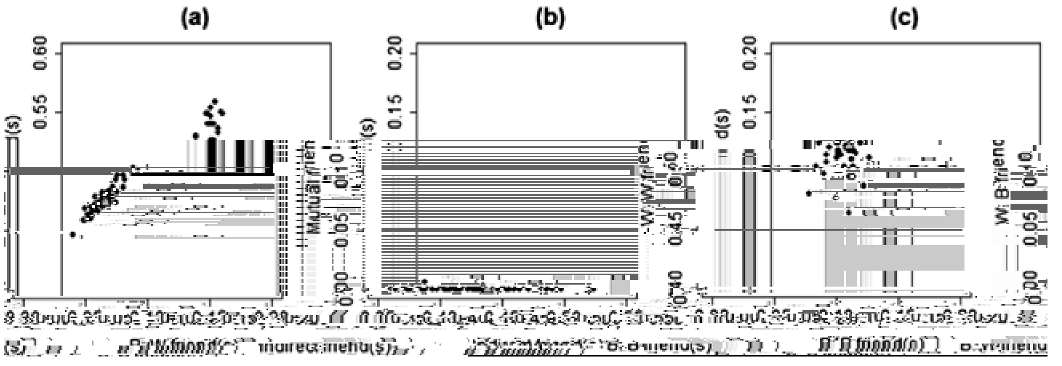


FIG E 7. E . *Notes*: F - : - (x: x ( )), - (x: y ( )), , . , .

(F 7). M A D.6 D.8 ( ) . , , (..,). M C M C (MCMC) (M A D.8). F 8. f_{WB} , , f_{BW} . A , , (.., $f_{BB} > f_{BW}$ $f_{WW} > f$ W B B B f BB f_{WW}

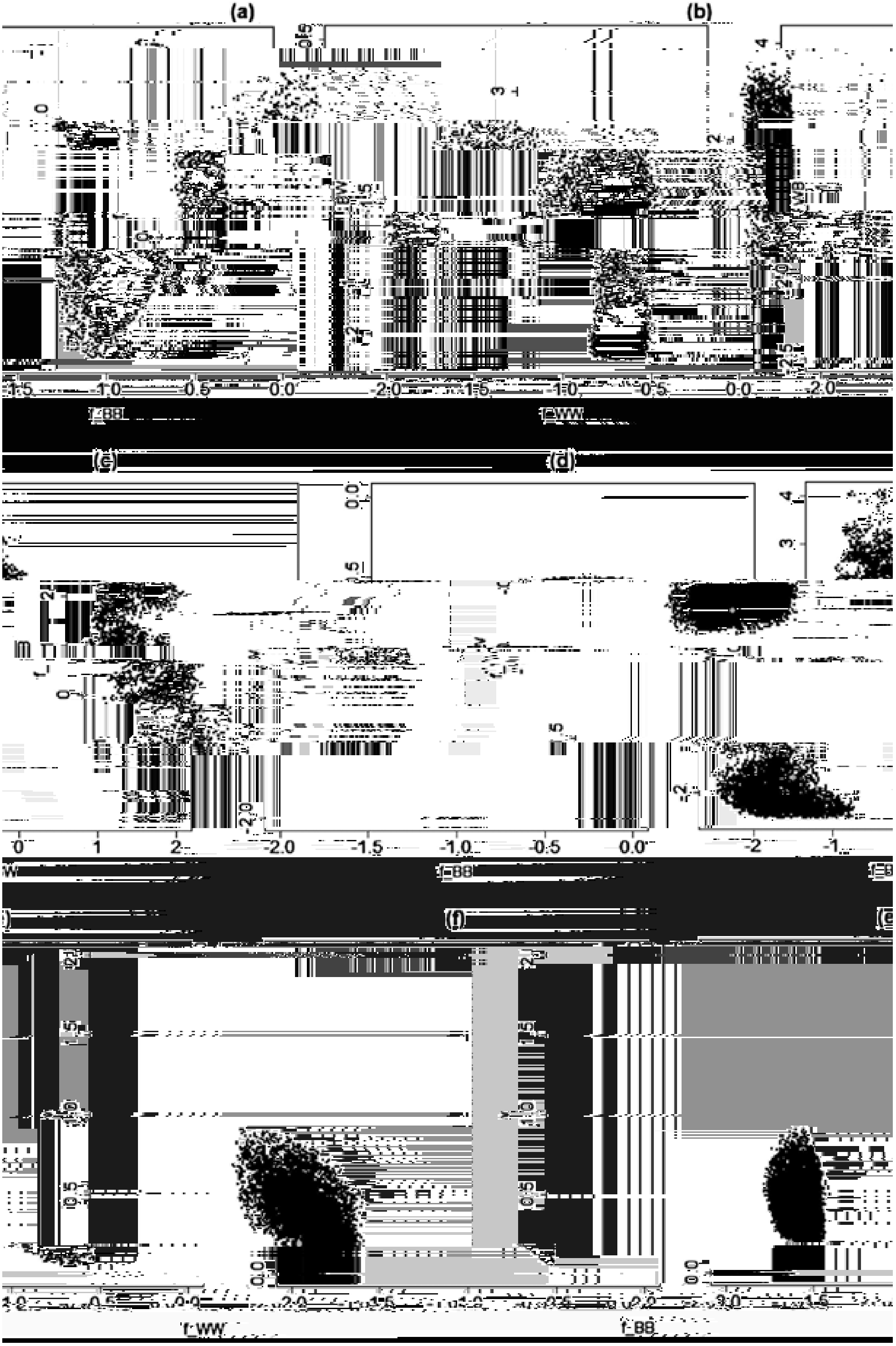


FIG E 8. L . Notes

8. C●NCL ●N

( ), . F J , -B , (2012), 5 8 . ● , . N

(. ., –) . E , . I , ,

A--ENDI A: - ●●F ●F HE● EM 1 AND 2

A.1. *Theorem 1*

- ●●F: G G, X, ε ( ), H $\alpha_H(t)$ H t. ; , $(\mu_{v_1(t)}/\mu)\sum_H P_{H|v_1(t)}\alpha_H(t) = \pi_t$. L G , ( ) D 1. H , i, (A v), u(A v; εi) ≥ u(A−l v; εi) l = 1 L. B , . H , $\alpha_H(t) > 0$ t H, C 1 . C 2 , (t s) L . L (t̄ s̄) s.) ■ .88.C 3H 645) s

$\mu_{v_1(s)} \sum_{\check{H}\in\mathcal{H}} P_{\check{H}|v_1(s)} \alpha_{\check{H}}(s) 1_{\bar{s}\in\check{H}}$

(t s) C 2 , . *Q.E.D.*

A.2. *Theorem 2*

F: C 2 .

$\mu_{v_1(\cdot)}$ $P_{\cdot|\cdot}$,

$$\mu_{v_1(t)}\mu_{v_1(s)} \sum_{\tilde{H}\in\mathcal{H}} \sum_{\check{H}\in\mathcal{H}} P_{\tilde{H}|v_1(t)} P_{\check{H}|v_1(s)} \alpha_{\tilde{H}}(t)\alpha_{\check{H}}(s) 1_{\bar{t}\in\tilde{H}} 1_{\bar{s}\in\check{H}} = 0$$

$$\Leftrightarrow \quad \sum_{\tilde{H}\in\mathcal{H}} \sum_{\check{H}\in\mathcal{H}} \alpha_{\tilde{H}}(t)\alpha_{\check{H}}(s) 1_{\bar{t}\in\tilde{H}} 1_{\bar{s}\in\check{H}} = 0$$

(3)

(π_t).

1 . F , C 1 (.,

$\{\alpha_H(t) : t \in H\}$, $\{\alpha_H(t)\})$. H , -

, . *Q.E.D.*

EFE ENCE

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B E A

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