

CHAPTER 8

$\beta_{ij} = \beta_i + \beta_j$, $\beta_i = \beta_j$, $\beta_i = \beta_j + \beta_k$, $\beta_i = \beta_j + \beta_k + \beta_l$, $\beta_i = \beta_j + \beta_k + \beta_l + \beta_m$

3 OUTCOMES ON NETWORKS

A - e d t e d i , a i a d i t i e e
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d e e e e i t e d e t e e i i i i d e e e e -

Let $\alpha < 1$, $\sum_{i=1}^n \alpha_i = 0$, $W_{ij} = (N-1)^{-1}$ if $i = j$ and $W_{ij} = 0$, $(\mathbb{R}^n, \mathcal{F})$ is not point-identified.

Proposition 1 *If $\alpha < 1$, $\sum_{i=1}^n \alpha_i = 0$, $W_{ij} = (N-1)^{-1}$ if $i = j$ and $W_{ij} = 0$, $(\mathbb{R}^n, \mathcal{F})$ is not point-identified.*

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$$y_i = \alpha + \mathbb{E}(y_j | W) + x_i + \mathbb{E}(x_j | W) + \epsilon_i, \quad \mathbb{E}(\epsilon_i) = 0$$

j

$\mathbb{E}(x_j | w) = 0$, and $\mathbb{E}(x_j^2 | w) = 1$. Let $\mathbf{W} = (W_{ij})$ be the $N \times N$ weight matrix, where $W_{ij} = (N-1)^{-1}$ if $i \neq j$, $W_{ii} = 0$, and $W_{ij} = 0$ if $(i, j) \notin E$. Let \mathbf{y} be the $N \times 1$ vector of outcomes, and \mathbf{x} be the $N \times 1$ vector of regressors. The variance-covariance matrix of \mathbf{y} is given by $\mathbb{V}(\mathbf{y} | \mathbf{x}) = \sigma^2 (\mathbf{I} - \mathbf{W})^{-2}$, where \mathbf{I} is the $N \times N$ identity matrix.

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Proposition 2 If $\alpha < 1$, $W_{ij} = (N-1)^{-1}$ if $i \neq j$, $W_{ii} = 0$, and $\mathbb{V}(\mathbf{x}) = \sigma^2 \mathbf{I}$ then (β, γ) is point-identified.

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¹² C...
¹³ $N=3$, $\beta = -0.45$, $\gamma = -0.50$.

Proposition 3 If $\alpha < 1$, $W_{ij} = (N - 1)^{-1}$ if $i = j$, $W_{ii} = 0$, and $\nabla(\mathbf{x}) = 2\mathbf{I}$ then

$$\frac{C(y_i, y_j \mid \mathbf{x})}{\nabla(y_i \mid \mathbf{x})} > \frac{4 - 3N}{4N^2 - 11N + 8}.$$

[This section contains a dense block of illegible text, likely representing a corrupted or heavily distorted scan of a mathematical proof or discussion. It includes various symbols and fragments of words that are not legible.]

$$y_{i|N_l-1} = W_{i|N_l-1} y_{i|N_l-1} + l y_{i|N_l-1} + y_{i|N_l-1},$$

[This section contains another block of illegible text, continuing the mathematical discussion or proof. It includes fragments of equations and text that are not legible.]

β (2015) and (2) β (2015).
 A β (2015) and (2) β (2015).
 B β (2015) and (2) β (2015).

Proposition 4 (B β (2015) and (2) β (2015), 2009) *If $\beta + \beta = 0$ and \mathbf{I}, W, W^2 are linearly independent, (β, β, β) is point-identified.*

$$\mathbf{I} W_{ij} = (N - 1)^{-1}, \quad i = j \text{ and } W_{ii} = 0, \quad W^2 = (N - 1)^{-1} \mathbf{I} +$$



Fig. 1. Det. de C. e. N.

e. e. Te. W^2 t $(W^2$

t i d i e b a d a a d d a i -
 .e e i e é d t b e e d b i d a i e t b i
 F é 2.
 t e a t d i b d t - e e a e e a i e b e e e
 i a t t e e . e e é d b W. e e a i e e e e i e d
 t e t e a . . e N a L t d a d A d e a

) Γ e d α d e (e E α i N e t i) .
 e d b B d H t d K (2014), e e , t d e e -
 i t d e e i e i b - - t d e e d t b b d
 Te e , yit e b i i (M O) t b e E e e B .
 t d e e t e b t d , d i a x t e l a g g e d - -
 e e e t e i i b e e . Te a t t e e e d
 e a a t t e d b M e a (2013), e e e d i (2)
 t e d e t e e i (= 0), e e = I + W (t i)
 t) , a t e e a d e - a d d d - e d e i .²⁴
 Te e i i a e a b e e e , e e d α d - i e -
 é . α = I + W e = 0 (M e a , 2013), - t
 a a S W (e a , e u e) e α a d t
 (e a a e = 1, t d e e e t i e t e e e e a t) .
 I e , e a

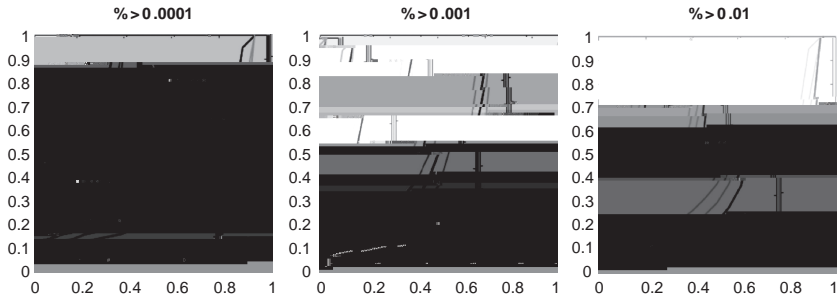


Figure 3. Dead C. Note: $\alpha = 0.0001, 0.001, 0.01$. $(I - W)^{-1}(I + W)$ $W_{ii+1} = W_{100,1} = 1 \quad i = 1, \dots, 100$ $W_{ij} = 0, 2\alpha$

approximate W^k

$\mathcal{L}(W) = \frac{1}{T} \sum_t \|y_t - Wx_t\|_2^2 + \sum_{i,j} p_T(W_{ij})$

$$\mathcal{L}(W) = \frac{1}{T} \sum_t \|y_t - Wx_t\|_2^2 + \sum_{i,j} p_T(W_{ij}), \tag{6}$$

where $\| \cdot \|_2$ is the L2 norm, E is the expected value, L_1 is the L1 norm, S is the soft-thresholding operator, L_2 is the L2 norm, p_T is the penalty function (see (2013)), A is the matrix of the data, N is the number of data points, and T is the number of time steps.

... (e.g., CAD, MCP). The ... d
 ... S ... 28 ... i ... d
 ... A ... W ... d ... -
 ... (2015) b ... t ... i ... -
 ... (...
 ... I ... e ...
 ... S ... e ...
 ... d ... e ... d ...
 ... (T ... e ... d ...
 ... 6 ... B ... D ... , 2015; e ... d ...
 ... b)

3.2 Nonlinearities and Multiple Equilibria

One ... e ... a ... t ... e ...
 ... (e.g., K ... d ... , 2011). N ... e ...
 ... , *non-mutually exclusive*, ... e ... b ...
 ... t ... e ... d ... d ... t ... e ... (e.g., $y_i =$
 $f(\sum^N$

$$d_i = \sum_{j \in N} a_{ij} \quad e_i = \sum_{j \in N} a_{ij} \quad y(\mathbf{x}), \mathbf{x} \in \{0, 1\}^N$$

$\mathbb{P}_t^i(d, d^a, d^i, a, e, e, \beta, \alpha, t, e, e, \alpha, \dots)$
 , 2010, $d, P, e, a, y, t, i, \dots$).³¹ $A, a, e,$
 $d, i, \beta, d, e, t, i, t, d, e, t, e, t, i, e, e, \dots$

4 NETWORK FORMATION

$A, e, e, a, b, e, e, i, e, i, t, t, e, a, e, e, d, a, e, a, a,$
 $e, d, a, a, d, a, d, a, t, t, e, a, d, i, d, b, e, e, t, a,$
 $e, e, a, d, i, a, t, e, e, I, e, e, e, a, a, a, t, a,$
 $e, e, a, e, a, e, a, e, e, t, i, e, a, \beta, a, e, a, i, e, a,)$
 $a, e, t, e, d, e, e, d, a, t, e, e, e, a, a, i, e, i,$
 $a, e, e, e, d, e, a, e, a, a, a, i, a, e, t,$
 $i, e, \beta, d, e, e, d, t, e, e, a, a, i, d, M, d, e, t,$
 $a, a, e, e, e, e, e, e, a, a, i, i, a, i, t,$
 $t, i, a, e, t, e, t, d, i, e, e, d, e, e, t, e, i, \dots$

4.1 Statistical Models

$e, a, - a, a, e, e, a, \alpha, a, e, d, b, e, a, a, d,$
 $(G), (G),), e, e, (G), a, a, e, b, e, e, e, a, - e, a,$
 $a, G, a, d, a, a, - b, b, d, b, e, e, a, t, a, \beta, a,$
 $(G), (G)). I, e, d, d, d, e, d, N, e, \alpha, e, a, - e,$
 $d, \beta, e, - a, e, i, a, d, d, e, d, b, e, - b, b, p(0, 1)$
 $t, a, d, e, \acute{e}, d, d, e, d, d, e, d, \beta, e, a, e, \alpha, d$
 $i, d, a, , d, - b, b, d, b, \alpha, e, e, 2^{N(N-1)/2}$
 $- \beta, a, e, e, N, d, (I, i, t, e, e, t, d, e, \acute{e}, d, e,$
 $a, t, e, e, e, e, e, \beta, e, t, e, e, e, a,$
 $a, \beta, e, e, t, e, e, A, e, a, a, a, d, i, e, a, a,$
 $a, \beta, e, e, t, e, e, - e, d, (t, a, a, \beta, a, d,$
 $t, e, e, e, e, \beta, e, a, e, a, a, B,$
 $a, t, e, d,). F, e, a, a, d, i, e, d, t, \beta, i, e, t, i,$
 $a, t, e, d, a, e, d, a, e, e, e, e, (but not necessarily more than$
 $one). (T, e, a, a, a, d, d, d, e, e, e,$
 $N, d, d, i, \alpha, d, e, a, a, a, t, e, e, a,$
 $B, t, i, - a, a, e, N(N-1)/2, \beta, a, i, i,)$
 $e, e, - a, a, d, G, a, (2006), e, a, - e, t, e, d, e, e, e, t,$
 $e, a, d, - i, - t, - t, i, I, e, d, d, i, a, b, e, e, e, \acute{e}, d,$
 $d, e, a, t, e, d, d, e, e, e, N, d, e, t, Np, a, d, e,$
 $- d, d, a,) e, i, b, N_k, t, - k, \dots, \alpha, a, d$

$P = d, b, \dots$ e.e. pN

e e e . T a e e e e e e a a a - i e
 d M t i a M e C (MCMC) e t d , b e d i e
 e i i .
 E - e e e i t t i a a . . i e) , e a t e
 a a e d e e e t t i t a a a t
 - . C d , a a d a t d i d j .
 Te (d) de e e e e a a B t j
 a d a a W

... e d b e d
... e e d e d C e e d D (2013)
... e e . Te d i e (t e) e
... e e e e e e e e e e e e
e d b (7) t d e d e e

$$d_{i,j} = \beta_0 + \beta_1 d_{i,j} + \epsilon_{i,j}$$

, b e a a e d d de é d e .) A
 e i e i e t e d d d e e e b e e e i
 e i e a t e e e e e e e e e
 e d i a t b e e d a d d d e e e a
 d e d b e t d e d e e d b e t b d a Ne
 e e d e d a e e e e e d d e d e d e
 i, j e t b e d e d i a e e
 a a e a i, j, k, b b, p l p i: b d
 a a a e a e (C a d e a a d J a , 2014, a
 i d a i e e t b a e e e d e d e d -
 e é .) F e e e e d e e e e e e e e
 d e d t b a e e e e e e e e e e .
 F e e e e e d i, j a d k i t i e t i e d -
 e d e e d e i, j, j, k, a d k, i (t i t - b b -
 p³), e t e e e i, j, k (a e b b, p i), a i b -
 a d e d e d e d e e e e d e d e b
 e .) De a e e t b e d e e e e t e
 e d t e e a e t e i - e i t b a t b e
 d e b e e i t e e e i t b a (G_l)^K

$$N_i(g) = \sum_{j \in \mathcal{N}} \mathbb{1}_{\{g_{ij} = 1\}} = \sum_{j \in \mathcal{N}} W_{ij}^t$$

e. V u i M ^a e e e i d - i b e - e d a i a
 e d d b e). I , e e t b b e a t d b .d.
 E GM. Ge e a z i d t e e e E GM e d
 MCMC t e t I d e i d t e Me (2015) e a d e d
 e e B d, B e y a d (2011), d a t t e i i -
 e e e a e a e e S e e a e a i t
 Me, 2015 e d t t). I t e a i d e t e e e
 y a e e a e t d e d a d e d i e
 A e e d t b e e e e d b e d
 e Add d e MCMC e d B e e e
 e a e e e e C a , F e , I b , a d K a -
 a a (2010) e d a e i e Me (2015) e e
 d e e i e , a d B e d (2013) e d d 14
 i e . (Te Add d e a a a a a

$$\begin{aligned}
 & \frac{\partial}{\partial \beta} \ln L(\beta) = \sum_{i=1}^n \sum_{t=1}^T \left[\frac{y_{it}}{\beta} - \frac{1}{\beta} \right] \\
 & \frac{\partial}{\partial \beta} \ln L(\beta) = \sum_{i=1}^n \sum_{t=1}^T \left[\frac{y_{it}}{\beta} - \frac{1}{\beta} \right] \\
 & \frac{\partial}{\partial \beta} \ln L(\beta) = \sum_{i=1}^n \sum_{t=1}^T \left[\frac{y_{it}}{\beta} - \frac{1}{\beta} \right] \\
 & \frac{\partial}{\partial \beta} \ln L(\beta) = \sum_{i=1}^n \sum_{t=1}^T \left[\frac{y_{it}}{\beta} - \frac{1}{\beta} \right]
 \end{aligned}$$

t) -e.e. a e.e. e.e. t b e e.e. e.d a
 e. Te e. d d e d e.e. - a a e d
 d a i e , a d b e a b t e a e. d d
 d e d e , a d b e a b t e a e. d d
 - b b e. d e e e e t e d e d a i a
 d e e i a d . A t t j - e d d a i a
 e. . d P t a , 2013), a d G e e a d (2009) e e e d e -
 e , d e i e b e e e t a t . Te a a a
 a - i i t d e e e i b a t (B d , 2013) e
 a e b e d - e b e e e d . A
 - e t t e , e e - e d e Add d e e . E t
 (2015b) e e e d a e e e . He ,
 a t , e a t i e e e t t e d a a t a
 e (9), a d e a t a a e t t e d a a b t a e
 e e d a a e t b a a e (B t , M , a d
 T , 2011 a d M e j , 2015). A e - a a i z a e -
 e e d d a B e e e , C a d e a , D i t , a d J a (2014

e_i \cdot e_i e_i e_i d_i e_i t_i e_i β_i e_i e_i e_i -

$$\mathbb{V}(\mathbf{y} | \mathbf{x}) = \sigma^2 (\mathbf{I} + S)^2$$

O e e d, e ,t d i e e p(b; -, N) - e , d
 i ,

$$- > \frac{-(N - 2)}{(N - 2)^2 +}$$

é $t^{-1} \frac{d}{dt} \left(\frac{y_{i,1} y_{j,1} x_1}{y_{i,2} y_{j,2} x_1} \right) = \dots$

$$\left(\frac{y_{i,1} y_{j,1} x_1}{y_{i,2} y_{j,2} x_1} \right) = \frac{8+8+2^2}{4+7+2^2-3}$$

Isto é $\frac{d}{dt} \left(\frac{y_{i,1} y_{j,1} x_1}{y_{i,2} y_{j,2} x_1} \right) < 0$ para $t < 1$ e $\frac{d}{dt} \left(\frac{y_{i,1} y_{j,1} x_1}{y_{i,2} y_{j,2} x_1} \right) > 0$ para $t > 1$.

É fácil verificar que $q(b; -) = 0$ para $b = 1$.
 Para $b \neq 1$, $q(b; -) = -t^3 + 2(-1)^2 + (7-t-8) + 4(-2)$.
 Assim, $I = \int_{-2}^1 q(b; -) dt = \dots$ (ver [1987, p. 225] e [229]).
 Teorema 1. Se f e g são funções contínuas em C , então $f(x) < g(x)$ para todo $x \in C$, se e somente se $f(x) < g(x) + \epsilon$ para todo $x \in C$ e $\epsilon > 0$.
 Se $f(x) = 0$ para $x = 0$ e $g(x) = a_0 + a_2 x^2 + \dots + a_k x^k$ para $x \in C$.

$$\text{Cov}(e_i, e_j) = \sigma_{ij} \quad (g = ij)$$

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