CHAPTER 8

[illegible]

2.1 Vertex Features

[illegible]

[1] [illegible]

[illegible] $g - 1$, [illegible] i [illegible] j [illegible] $(i, i_1, \ldots, i_{K-1}, i_K, j)$ [illegible] i_k, i_{k+1} [illegible] $\mathcal{E}_g$. A [illegible] $i_1 = i_K$, [illegible] g [illegible] $\mathcal{E}_g$ [illegible]

[illegible]

[illegible]

3 OUTCOMES ON NETWORKS

A [illegible]

[illegible] $\beta < 1$, [illegible] $\mathbf{I} - \beta W$ [illegible] $(\alpha, \beta, \eta, \gamma)$ [illegible]:

Proposition 1 *If* $\beta < 1$, $\eta\beta + \gamma = 0$, $W_{ij} = (N-1)^{-1}$ *if* $i = j$ *and* $W_{ii} = 0$, $(\alpha, \beta, \eta, \gamma)$ *is not point-identified.*

[illegible] (1993) [illegible] P [illegible] 1 [illegible] B [illegible], D [illegible] F [illegible] (2009). [illegible] K [illegible], P [illegible] (2006) [illegible] (1993). I [illegible] (1), M [illegible]

$$y_i = \alpha + \beta \mathbb{E}(y_j \; w) + \eta x_i + \gamma \mathbb{E}(x_j \; w) + \;_i, \quad \mathbb{E}(\quad\quad =$$

j

[illegible] E

[illegible] x_i. A [illegible] M[illegible] (1993) [illegible] (p. 535). [illegible] P[illegible] 1 [illegible] x, [illegible] $\mathbb{V}(\mathbb{E}(x_j\ w)) = 0$, [illegible]

A[illegible] (2) [illegible]. I [illegible] $\mathbb{V}(\ \mathbf{x}) = \sigma^2\mathbf{I}$, [illegible]

$$\mathbb{V}(\mathbf{y}\ \mathbf{x}) = \sigma^2(\mathbf{I} - \beta W)^{-2}.$$

[illegible] β [illegible] $W_{ij} = (N-1)^{-1}$ [illegible] $i = j$ [illegible] $W_{ii} = 0$. I[illegible] M[illegible] (2001) [illegible] (F[illegible], 1966; B[illegible] P[illegible], 1986; H[illegible], N[illegible] T[illegible], 1987). B[illegible], I [illegible] N [illegible].[12]

Proposition 2 *If* $\beta < 1$, $W_{ij} = (N-1)^{-1}$ *if* $i = j$, $W_{ii} = 0$, *and* $\mathbb{V}(\ \mathbf{x}) = \sigma^2\mathbf{I}$ *then* $(\alpha, \beta, \eta, \gamma)$ *is point-identified.*

I[illegible] N [illegible] 3. T[illegible]: I [illegible] i' [illegible] β [illegible] j. I [illegible] k [illegible] i' [illegible]. T[illegible] k [illegible] j' [illegible] i' [illegible]. (O[illegible] $N = 2$ [illegible] -1. T[illegible] $N > 2$.[13]) A[illegible] ([illegible]), P[illegible] 2 [illegible]. T[illegible]

[12] C[illegible]

[13] [illegible] $N = 3$, [illegible] P[illegible] 3 [illegible] -0.45. F[illegible] -0.50.

Proposition 3 *If* $\beta\ < 1$, $W_{ij} = (N-1)^{-1}$ *if* $i = j$, $W_{ii} = 0$, *and* $\mathbb{V}(\ \mathbf{x}) = \sigma^2\mathbf{I}$ *then*

$$\frac{\mathbb{C}(y_i, y_j\ \mathbf{x})}{\mathbb{V}(y_i\ \mathbf{x})} > \frac{4-3N}{4N^2-11N+8}.$$

[illegible] $l = 1, \ldots, L$, [illegible] α_l), [illegible] d'H[illegible] (2009) [illegible] $\mathbb{V}(\ \mathbf{x}) = \sigma^2\mathbf{I}$ [illegible] *at least* [illegible] 3.2). [illegible] (2013) [illegible] $\mathbb{V}(\ \mathbf{x}) = \sigma^2\mathbf{I} + \sigma\ (W + W\ )$. [illegible] W [illegible] (2009) [illegible]

[illegible] (2) [illegible] 1996 [illegible] (2008), [illegible] $l = 1, \ldots, L$ [illegible]

$$\mathbf{y}_{l\,N_l\ 1} = \gamma W_{l\,N_l\ N_l}\ {}_{l\,N_l\ 1} + \alpha_l \mathbf{1}_{N_l\ 1} + {}_{l\,N_l\ 1},$$

[illegible] N_l [illegible] l, $W_{ij,l} = (N_l-1)^{-1}$ [illegible] $i = j$ [illegible] $W_{ii,l} = 0$, [illegible] α_l [illegible] .[14] (A[illegible] 2003.) [illegible] $_{i,l}$, [illegible] $\sum_{j=i}\ {}_{j,l}/(N_l-1)$, [illegible] α_l. [illegible]

[illegible] (2015) [illegible] (2) [illegible] 5).

A [illegible] (2) [illegible] B[illegible], D[illegible], [illegible] F[illegible] (2009):

Proposition 4 (B[illegible], D[illegible], [illegible] F[illegible], 2009) *If* $\eta\beta + \gamma = 0$ *and* $\mathbf{I}$, W, W^2 *are linearly independent,* $(\alpha, \beta, \eta, \gamma)$ *is point-identified.*

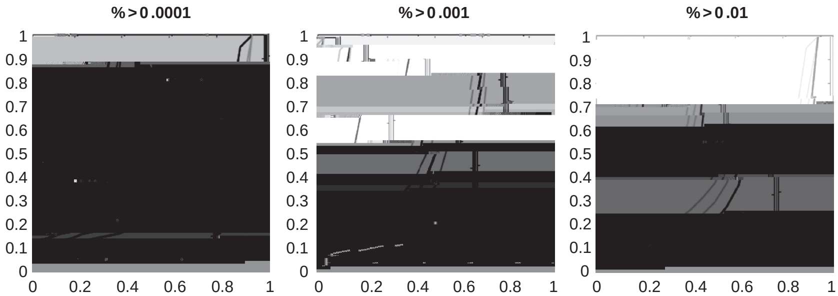
I[illegible] $W_{ij} = (N-1)^{-1}$ [illegible] $i = j$ [illegible] $W_{ii} = 0$, $W^2 = (N-1)^{-1}\mathbf{I} +$

F te 1 De é d C é N .

e e . Te a W^2 t a $(W^2$

[illegible]

[illegible]



F[illegible] 3 [illegible] De[illegible] C[illegible]. *Note:* [illegible] $(\mathbf{I} - \beta W)^{-1}(\mathbf{I} + W)$ [illegible] 0.0001, 0.001 [illegible] 0.01, [illegible] β. [illegible] W [illegible] 100 100 [illegible] $(N = 100)$, [illegible] $W_{i i+1} = W_{100,1} = 1$ [illegible] $i = 1, \ldots, 100$ [illegible] W [illegible] 1 [illegible] $\mathbf{I} + W$, [illegible] $\beta = 0$, [illegible] 2 [illegible].

[illegible] d, [illegible]. A [illegible] *approximate* [illegible] (i, j) [illegible] W^k [illegible],

[illegible] W [illegible]. I [illegible] LASSO [illegible] E [illegible] N [illegible]. ([illegible] β.)

[illegible] W [illegible] Lam and Souza (2013), [illegible] W. [illegible]

$$\min_{(W,\beta,\delta,\gamma)} \frac{1}{T}\sum_t \|\mathbf{y}_t - \alpha - \beta W\mathbf{y}_t - \eta\mathbf{x}_t - \gamma W\mathbf{x}_t\|_2^2 + \lambda \sum_{i=j} p_T(W_{ij}), \tag{6}$$

[illegible] $\|\ \|_2$ [illegible] E [illegible] L_1 [illegible] W. [illegible] y [illegible] Lam and Souza (2013) [illegible] it [illegible] (see [illegible] A2). ([illegible] N [illegible] T

[illegible] (CAD MCP). [illegible][28]

[illegible] (2015) [illegible] 2015; [illegible]

3.2 Nonlinearities and Multiple Equilibria

[illegible] 2011). [illegible] *non-mutually exclusive*, [illegible] (e.g., $y_i = f\left(\sum^N\right.$ [illegible]

d b t e e a t e $\mathbf{y}(\mathbf{x}), \mathbf{x}$ $0, 1$ ^{N}

[illegible] 2010, [illegible] [31] [illegible]

4 NETWORK FORMATION

A [illegible]. I [illegible]. M [illegible].

4.1 Statistical Models

[illegible] $(\mathcal{G}, \sigma(\mathcal{G}), \ )$, [illegible] $\sigma(\mathcal{G})$ [illegible] σ- [illegible] $\mathcal{G}$, [illegible] $(\mathcal{G}, \sigma(\mathcal{G}))$. I [illegible] N [illegible] p [illegible] $(0, 1)$ [illegible] $2^{N(N-1)/2}$ [illegible] N [illegible]. (I [illegible].) A [illegible]. F [illegible] (*but not necessarily more than one*). (T[illegible] N [illegible] $N(N-1)/2$ [illegible].)

[illegible] (2006), [illegible]. I [illegible] N [illegible] Np, [illegible] k [illegible] N_k

a P d b t v a a e e pN

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

A PROOFS

A.1 Proof of Proposition 2

I[illegible] $\mathbb{V}(\cdot|\mathbf{x}) = \sigma^2\mathbf{I}$, [illegible] $\mathbb{V}(\mathbf{y}|\mathbf{x}) = \sigma^2(\mathbf{I}-\beta W)^{-2}$. [illegible] $|\beta| < 1$ and W [illegible] (-) [illegible]

$$(\mathbf{I}-\beta W)^{-1} = \mathbf{I}+\beta W+\beta^2 W^2+\dots$$

I[illegible] $W^k, k = 1, 2, \dots$ [illegible] $(W^k)_{ii} = a_{k-1}$ and [illegible] $(W^k)_{ij} = a_k, i \neq j$, [illegible]

$$a_0 = 0, \qquad a_{-1} = 1 \quad \text{and} \quad a_k = (a_{k-2} + a_{k-1}(N-2))/(N-1).$$

[illegible] $S = \sum_{k=1} \beta$

$\mathbb{V}(\mathbf{y}\ \mathbf{x}) = \sigma^2(\mathbf{I} + S)^2,$

O e e d, e t d e é $p(b; \overline{\kappa}, N)$ e , d

$$\overline{\kappa} > \frac{-(N-2)}{(N-2)^2 +}$$

$$\psi(\beta) = \frac{\mathbb{C}(y_{i,1}, y_{j,1} \mathbf{x}_1)}{\mathbb{C}(y_{i,2}, y_{j,2} \mathbf{x}_1)} = \frac{8 + 8\beta + 2\beta^2}{4 + 7\beta + 2\beta^2 - \beta^3}.$$

I … $\psi(\beta) < 0$ … $\beta < 1$ … $\psi(\beta) > \psi(1) = 1.5$.

… $\overline{\psi}$ … β … $q(b;\overline{\psi}) = 0$ … $q(b;\overline{\psi}) = -\overline{\psi}\beta^3 + 2(\overline{\psi} - 1)\beta^2 + (7\overline{\psi} - 8)\beta + 4(\overline{\psi} - 2)$. … 1 … I … (… 1987, … 225 … 229). … I … f … g … C … $g(x) < f(x)$ … C, … f … $f + g$ … C, … $f(x) = a_1x$ (… $f(x) = 0$ … $x = 0$) … $g(x) = a_0 + a_2x^2 + \ldots + a_Kx^K$ …

( ) T ee e [illegible] $(g = ij,$

A[illegible], E., A. H[illegible], J. [illegible], and C. [illegible] (2011): [illegible] P[illegible], *Proceedings of the National Academy of Sciences*, 108(13), 5199–202.

B[illegible], A. (2013): [illegible] E[illegible] N[illegible]: The [illegible] and P[illegible], [illegible] P[illegible] [illegible].

B[illegible], [illegible], and G[illegible] (2000): [illegible] A N[illegible] M[illegible] N[illegible] F[illegible], *Econometrica*, 68(5), 1181–229.

B[illegible], C., A. C[illegible]-A[illegible], and [illegible] (2006): [illegible] N[illegible] [illegible] d: The K[illegible] P[illegible], *Econometrica*, 74, 1403–17.

B[illegible], A., A. C[illegible] and [illegible], E. D[illegible], and M. J[illegible] (2014): [illegible] G[illegible]: I[illegible] C[illegible] I[illegible] N[illegible], MIT P[illegible].

B[illegible], A., and [illegible]. A[illegible] (1999): [illegible] N[illegible], *Science*, 286, 509–12.

B[illegible], P., and D. P[illegible] (1986): [illegible] L[illegible] M[illegible] C[illegible], *Journal of Econometrics*, 31(2), [illegible]. 179–208.

B[illegible], A., [illegible], and C. H[illegible] (2013): [illegible] M[illegible] [illegible] 348.8(4)-250 L82

B ye , M. A., a d E. Ca ll a (2001): . de e Ba B ’ P e A :

Christakis, N., J. Fowler, G. Imbens, and K. Kalyanaraman (2010): "An Empirical Model for Strategic Network Formation," Harvard University Working Paper.

Chwe, M. (2000): "Communication and Coordination in Social Networks," *Review of Economic Studies*, 67, 1–16.

Ciliberto, F., C. Murry, and E. Tamer (2015): "[illegible] Market Power [illegible] Markets

(1960): [illegible] *Publications of the Mathematical Institute of the Hungarian Academy of Sciences*, 5, 17 61.

F[illegible], M., and [illegible] (2003): [illegible] *Journal of Development Economics*, 71, 261 87.

F[illegible], J., and [illegible] L. (2001): [illegible] *Journal of the American Statistical Association*, 96(456), 1348 60.

[illegible], I., and M. [illegible] (2014): [illegible] B[illegible] Press.

F[illegible], F. (1966): *The Identification Problem in Econometrics*. N[illegible]: M[illegible] H[illegible].

F[illegible], B., G. L[illegible], and M.-C. [illegible] (2007): [illegible] *Journal of Public Economics*, 91(11-12), 2089 112.

F[illegible], O., and D. [illegible] (1986): [illegible] *Journal of American Statistical Association*, 81, 832 42.

G[illegible], and G. [illegible] (2012): [illegible] *Journal of Public Economics*, 96, 485 94.

G[illegible], A., [illegible], and D. [illegible] (2013): [illegible]

G[illegible], E., and A. [illegible] (2014): [illegible]

G[illegible], C. J. (2009): [illegible] *Electronic Journal of Statistics*, 3, 259 89.

G[illegible], E. (1959): [illegible] *Annals of Mathematical Statistics*, 30(4), 1141 44.

G[illegible], J., K. K[illegible], [illegible] M. J. N[illegible] (2009): [illegible] *American Economic Journal: Applied Economics*, 1(4), 34–68.

H[illegible], M. (2003): [illegible]

H[illegible], M., [illegible] K. Ge[illegible] (2010): [illegible] *The Annals of Applied Statistics*, 4(1), 5–25.

H[illegible], J., [illegible] (1987): [illegible] *Econometrica*, 55(4), 849–74.

H[illegible], P. (2005): [illegible] *Journal of the American Statistical Association*, 100, 286–95.

H[illegible], P., A. [illegible], [illegible] M. H[illegible] (2002): [illegible] *Journal of the American Statistical Association*, 97, 1090–98.

H[illegible], P. [illegible] (1981): [illegible] *Journal of the American Statistical Association*, 76, 33–65.

H[illegible], H., [illegible] J. [illegible] (2014): [illegible]

H[illegible], B. E., [illegible] A. [illegible] (2010): [illegible] *Review of Economic Studies*, 77, 1138–63.

H[illegible], J., [illegible] A. M[illegible] (1993): [illegible] *Review of Economic Studies*, 60(3), 497–529.

H[illegible], C.-[illegible] L.-F. Lee (2013): [illegible]

H[illegible], D. [illegible], P. N. K[illegible] M. [illegible] (2012): [illegible] *Journal of Computational and Graphical Statistics*, 21(4), 856–82.

J[illegible], M. (2009): *Social and Economic Networks*. P[illegible]

J[illegible], M., [illegible] B. [illegible] (2007): [illegible] *American Economic Review*, 97(3), 890–915.

J[illegible], M., [illegible] A. [illegible] (2002): [illegible] *Journal of Economic Theory*, 106, 265–95.

J[illegible], M., [illegible] A. [illegible] (1996): [illegible] *Journal of Economic Theory*, 71(1), 44–74.

J[illegible] (2012): [illegible]

J[illegible], M. I. (2004): [illegible]

K[illegible], H. H., I. [illegible] P[illegible] (2006): [illegible] *Journal of Regional Science*, 46(3), 507 [illegible]

K[illegible], B., [illegible] d E. [illegible] (2011): [illegible] L[illegible] M[illegible] N[illegible] P[illegible].

K[illegible], E. (2009): *Statistical Analysis of Network Data*. [illegible]

K[illegible], D., [illegible] d N. F[illegible] (2009): *Probabilistic Graphical Models: Principles and Techniques*. MIT P[illegible].

K[illegible], A. A[illegible], [illegible] d K. [illegible] (2014): [illegible] C[illegible] P[illegible].

K[illegible], J. H., G. [illegible], [illegible] d P. P[illegible] (2010): [illegible] *Statistical Methodology*, 7, 366 [illegible] 84.

K[illegible], [illegible] d D. M[illegible] (2001): [illegible] *American Economic Review*, 91(3), 485 [illegible] 508.

L[illegible], C., [illegible] d P. C. [illegible] (2013): [illegible] H[illegible] -D[illegible] M[illegible] L[illegible] P[illegible].

(2014): [illegible] M[illegible] d F[illegible] d E[illegible] L[illegible] P[illegible].

L[illegible], [illegible] d E. [illegible] d (2015): [illegible] P[illegible].

L[illegible], N. (2015): [illegible] *Quantitative Economics*, 6, 49 [illegible] 83.

L[illegible], L.-F. (2007): [illegible] *Journal of Econometrics*, 140(2), 333 [illegible] 74.

L[illegible], L.-F., [illegible]

M[illegible], C. F. (1993): [illegible] *The Review of Economic Studies*, 60(3), 531[illegible]42.

(2013): [illegible] *Econometrics Journal*, 16(1), [illegible]1[illegible]23.

M[illegible], C. F., [illegible] J. M[illegible] (2003): [illegible] *Journal of the European Economic Association*, 1(1), 181[illegible]211.

M[illegible], M. A. (2015): [illegible]